Experiments on convection of isolated masses of buoyant fluid

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SUMMARY

Isolated masses of buoyant fluid were released in a water tank. Their width, $2r$, and the distance travelled, z, were measured as functions of time and were found to follow roughly the laws

$$r = nz, \qquad w = C(g\overline{B}r)^{1/2},$$

where w is the vertical velocity, $\overline{B}$ the mean buoyancy, and n and C are constants. These equations are predicted by dimensional analysis, assuming viscosity to be negligible, and the constants appear to be independent of the Reynolds number. It is found that $C \doteqdot 1{\cdot}2$, and n is in the neighbourhood of 4.

Since the Froude number relating the buoyancy and inertia forces is the same as for isolated masses of buoyant air in the atmosphere, it is concluded that the constants will have the same value in this latter case. This is confirmed roughly by observation of cumulus cloud towers.

Some of the characteristics of the motion observed in the experiments are described and comparison is made with vortex rings.

1. THE NATURE OF THE PROBLEM

It was desired to study motion produced in a fluid by buoyancy forces alone far from solid boundaries, in order to simulate buoyant convection in the atmosphere. From the behaviour of cumulus clouds and the experience of glider pilots over many years, it appears that convection, by which in this paper we mean convection produced by buoyancy forces, consists largely of more or less isolated masses of buoyant air rising into and mixing with their surroundings from intermittent sources on the ground or within clouds (see, for example, Ludlam & Scorer 1953; Welch, Welch & Irving 1955).

For simplicity in the experiments, masses of heavy solutions were released from rest at the top of a water tank, and their growth downwards was observed photographically. If the volume at release is sufficiently small compared with its subsequent volume and no impulse is applied, the subsequent growth can be expected to be largely independent of the initial configuration of the substance released. In such case the factors determining the motion are so few that a very simple dimensional analysis can be applied. The method has been discussed by Batchelor (1954), and experiments of this kind have been carried out by Morton, Taylor & Turner (1956), and Scorer & Ronne (1956). The motion is so complicated that no way was seen whereby the simplest dimensional analysis could be improved upon, and so the numerical coefficients relating a bulk dimension of the cloud of buoyant fluid to its buoyancy and vertical velocity were measured, with a view to applying them to the atmosphere.

2. The characteristics of 'thermals'

Following the usage of glider pilots, the buoyant masses of fluid are called 'thermals' herein. Over the advancing front of the thermal, mixing with the fluid ahead takes place. Inside and at the rear the fluid circulates rather as in a vortex ring, the motion being relatively smooth. As a result of the mixing the volume continuously grows, and, although the vertical

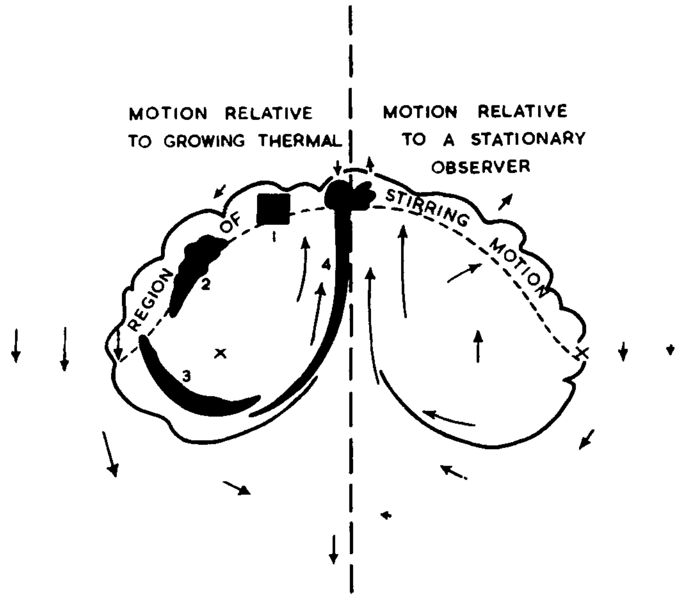

Figure 4. Approximate distribution of velocity in a typical thermal. The black area illustrates successive positions of a portion of the thermal, into which mixing has just occurred, beginning with the square.

momentum steadily increases through action of the buoyancy force, the velocity decreases on account of the incorporation of exterior fluid. Examples are seen in figures 1, 2 and 3 (plates 1 and 2). The pattern of motion is represented in figure 4. An idea of the exterior motion is gained from figure 3 (plate 2).

The thermals were released from a thin hemispherical copper cup (seen in figure 3, plate 2) pivoted about a horizontal axis, the level of the fluid inside being made the same as outside before release so as to avoid any

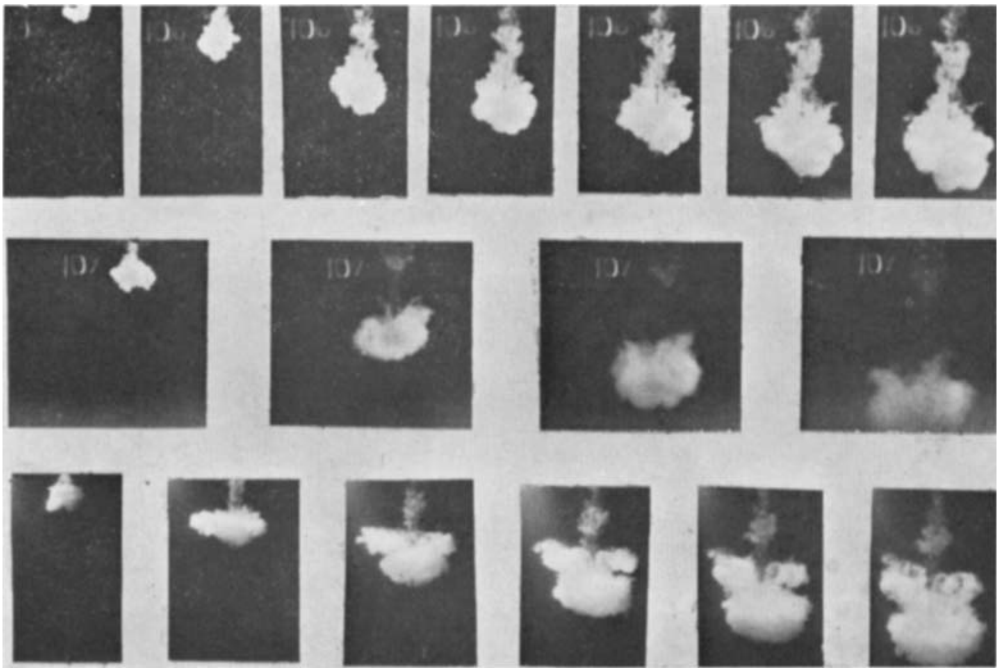

Figure 1. Series of pictures taken from cine film showing the growth of thermals in a water tank, made visible by white precipitate. (Experimental numbers 106, 107, 111). No. 107 is unusually well formed.

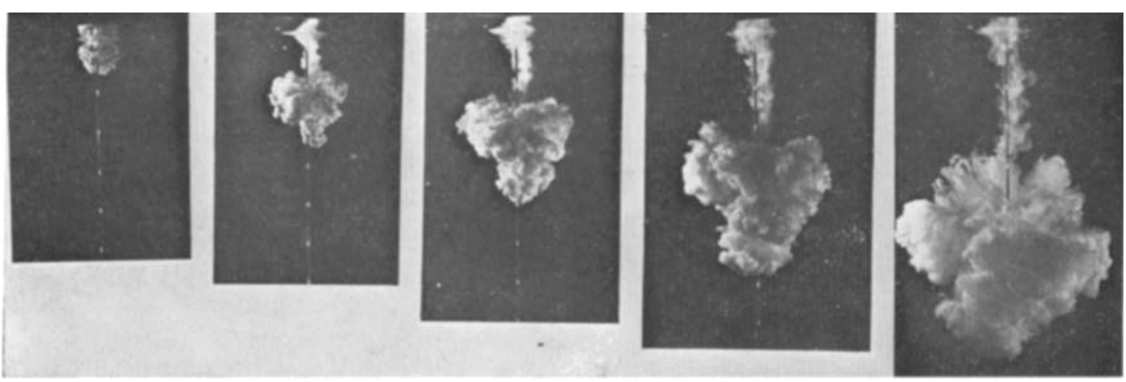

Figure 2. Pictures illustrating how a shape assumed early in the life of a thermal may persist while the volume increases several times. This may be contrasted with no. 111 of figure 1 in which the protuberances are shed. Note the ' cauliflower ' nature of the surface. No shearing motion is visible in these protuberances.

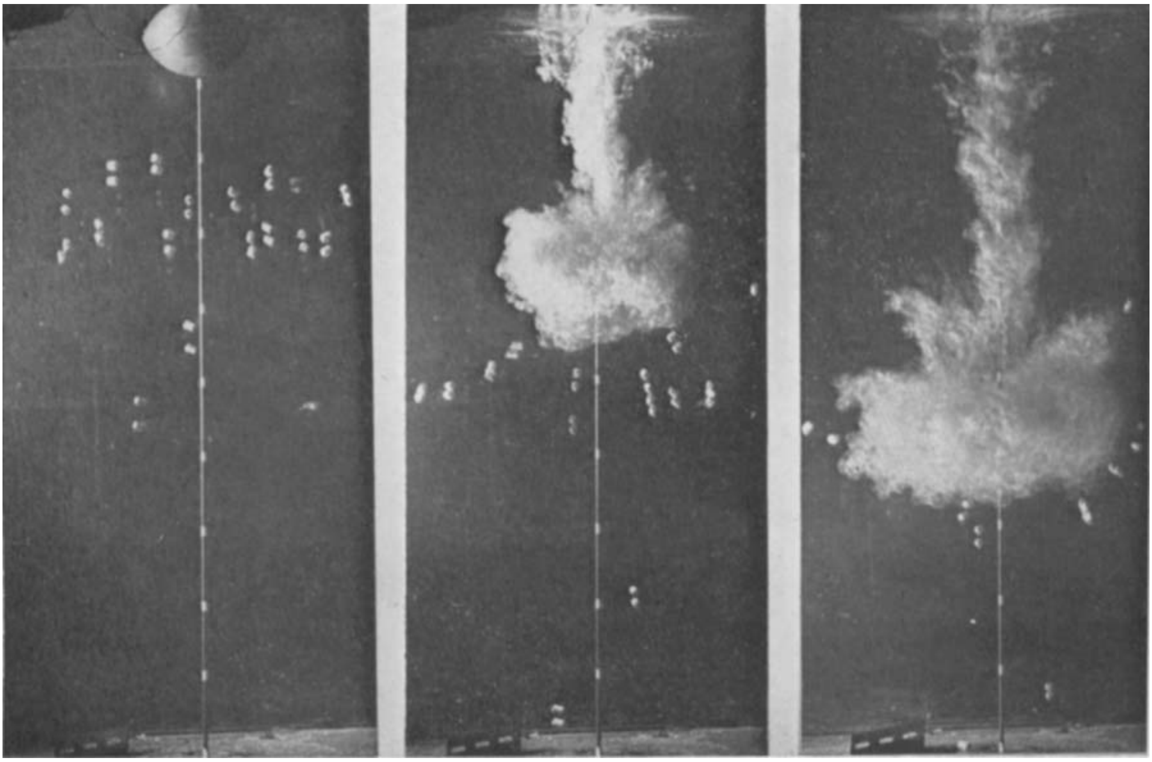

Figure 3. In each picture there are two exposures about $\frac{1}{2}$ sec apart. The first picture shows the free rate of sink of some slowly sinking particles. In the second and third pictures an idea of the velocity field outside the thermal can be gained. The particles immediately in front of the thermal are displaced downwards as much as the faster sinking ones lower down, while those at the side are in an upcurrent. Those that have disappeared have entered the advancing ' front ' of the thermal, not the rear. The markings on the vertical rod are at 10 cm intervals.

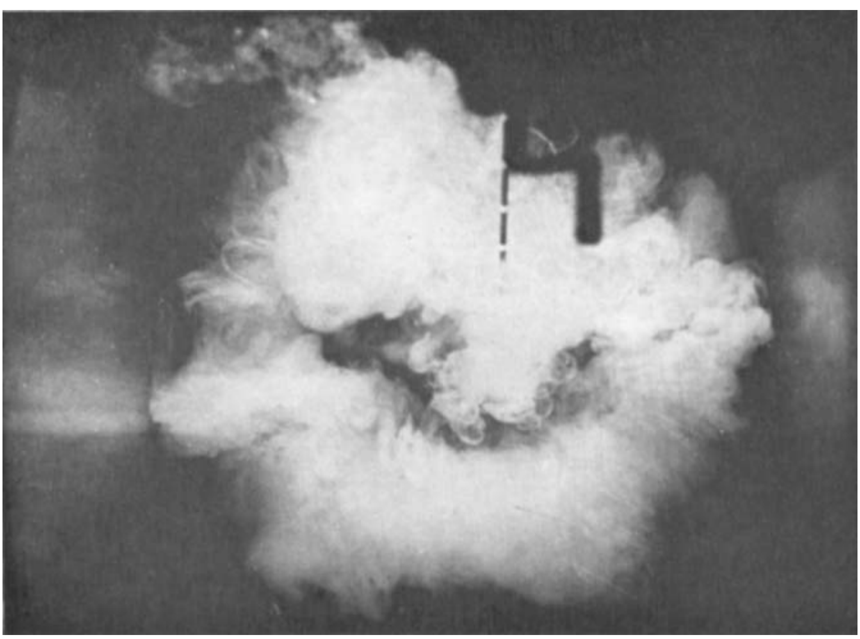

Figure 9. View of a thermal from above showing the hollowed-out rear.

initial impulse. The cup was turned over quickly by hand, and a negligible amount of motion was produced thereby. A dense white precipitate was used to make the buoyant fluid visible. Immediately after release the front surface of the thermal became covered with protuberances and the volume began to increase. After the main mass escaped from the surface there was often a stem left behind. But this was very dilute and had negligible velocity in it, and was therefore assumed to contain a negligible fraction of the original material. This stem is thought to be a kind of 'splash' produced as the original material becomes rearranged into the configuration which it ultimately assumes.

The motion consists initially of an acceleration. After the thermal has travelled about 1·5 diameters deceleration begins, and the measurements were concerned with the subsequent motion.

In order that simple dimensional analysis should be applicable it is necessary to suppose that the thermal behaves as it if originated at a virtual point origin. Therefore, in order to allow location of this origin as closely as possible, the same cup (radius 3 in.) was used in nearly all the experiments and was filled to approximately the same depth each time. Thus at least the geometry of the initial conditions was approximately the same, and the origin was assumed to be in the same position in all these cases. Some other cases were included when they were susceptible of easy measurement, that is, when the thermals grew with reasonable symmetry.

The density differences were kept small, and were initially less than 15%. If this is not done two difficulties arise. First, the motion may be different according to whether the heavier fluid is inside or outside (downward or upward motion), and, second, it may vary according to the magnitude of the density differences. During the measured part of the thermals' lives the density differences were mostly much less than 5%, and were reduced to around 0·1% towards the end of each experiment.

Some thermals grew in a grossly asymmetrical manner. This is thought to be due to internal motions of the thermal, or occasionally in the tank, at the moment of release. These cases were rejected.

Occasionally odd characteristics of shape seen in the early stages were retained almost throughout the observed life of a thermal (see figure 2, plate 1). In particular the angle subtended at the point of origin tended to remain the same, though it varied somewhat from thermal to thermal.

3. Simple dimensional analysis

The only factors which can determine the velocities are the buoyancy forces and the size. The local buoyancy force on unit mass of fluid is

$$g\frac{\rho_0-\rho_i}{\rho_0} = gB, \tag{1}$$

ρ_i and ρ_0 being the interior and exterior densities. The density ratio B varies within the thermal because the dilution varies, and so the mean value $\bar{B}$ will be used, the distribution of B being at present unknown. If r is the radius of the largest horizontal section, the vertical velocity of the front of the thermal must then be of the form

$$w^2 = C^2 g\overline{B}r \tag{2}$$

from dimensional considerations. C^2 is a Froude number, being a ratio between inertia forces and buoyancy forces.

The same formula can likewise be derived for the rise of a bubble of buoyant fluid which will not mix the surroundings. One example of this is the formula for the rise of bubbles of air through water, derived by Davies & Taylor (1950). There are some interesting differences. The buoyancy B in that case is virtually unity, but it is also constant, and for this reason it is not necessary to assume that B is small in the case of immiscible fluids. The drag is here due to the formation of a wake, so that the buoyant energy is converted into energy of turbulent motion of the surroundings; in our case there is no wake, but the 'drag' is due to the continuous incorporation of outside fluid.

If z is the height of the foremost part of the thermal the two velocities dz/dt and dr/dt are proportional to all other velocities, and so

$$z = nr \tag{3}$$

if the origin of z is suitably chosen. This, the virtual origin, is at the apex of the cone swept out by the largest horizontal section. n has to be determined by experiment, and, like C, must be the same for all thermals, according to the assumptions made. Since only one representative velocity exists (which is equivalent to saying that there is no Froude number other than C^2), the motion is similar at all stages in the life of a thermal and all thermals are similar.

The volume V of the thermal, which is seen in the experiments to possess a clearly defined outline, is given by

$$V = mr^3, \tag{4}$$

where m is another number to be determined by experiment. The total buoyancy of the thermal is constant, so that if suffix 0 denotes the values at the moment of release, when ρ_i is uniform,

$$g\overline{B}mr^3 = gB_0 V_0. \tag{5}$$

In the experiments B_0 and V_0 were measured before the release of the thermal.

Writing $w = dz/dt$ and integrating (2), we obtain

$$kz^2 = t, \tag{6}$$

where

$$k = m^{1/2}/2nC(gB_0 V_0)^{1/2}, \tag{7}$$

the origin of t being suitably chosen.

The objective in the experiments was first to confirm the relation (6) for each thermal separately, and then (7) for a variety of values of $gB_0 V_0$, the total weight (deficiency or excess) of the thermal.

4. The effect of viscosity

In previous work on experiments and analysis of this kind (Batchelor 1954; Morton, Taylor & Turner 1956), the motion has been described as 'fully turbulent', and it has been assumed that the viscous forces are negligible in comparison with the inertia forces (including the eddy stresses) and the buoyancy forces. In the case of a conical plume from a maintained source of buoyancy, the fact that the plume is conical confirms the correctness of this assumption because the Reynolds number varies up the plume. Furthermore, the motion is quite evidently turbulent throughout the volume of the plume. However, when small drops of coloured fluid are let fall in clear fluid they do not develop turbulent motion but retain a smooth outline. In an individual thermal from a source which is not maintained, the Reynolds number wr/ν is equal to $1/2k\nu$, which is the same throughout the life of a thermal; so that it would in any case be expected to grow along a cone and retain a constant shape even if viscous forces were important. But the Reynolds number varies from one thermal to another, and so if (7) is confirmed we can assume that variations in Reynolds number make no difference and that viscous forces are negligible.

In the case of thermals in cumulus clouds (see § 7 below) the Reynolds number is about one thousand times greater than in these experiments. The constant C is roughly the same, and so the assumption that viscosity can be neglected is further justified.

5. Experimental results

The thermals were released into a tank with horizontal section 2 ft. × 4 ft. The thermals traversed the depth of $3\frac{1}{2}$ ft. in between 5 and 30 seconds according to the weight excess.

The virtual origin was determined roughly by plotting r against z, extrapolating backwards, and taking $z = 0$ where $r = 0$. Variations in the position of the virtual origin by 3–4 cm had a negligible effect on the value of k obtained when z^2 was plotted against t, and so the origin was taken to have the same position relative to the bottom point of the cup in all cases in which V_0 was nearly the same. In a few cases V_0 was substantially different and for those the origin was determined individually.

The crudeness with which the measurements have to be made must be mentioned. Successive outlines of four typical thermals are shown in figure 5. With them are the plots of z^2 against t and the straight lines drawn to determine k. As expected, the points deviate from the line in the early part of the life while the thermal grows into its ultimate shape. Some of the thermals grew somewhat asymmetrically or irregularly but nevertheless yielded fairly definite values of k. The results are given in table 1.

No qualitative difference in the motion was evident when the motion was upwards. A fluorescent dye was used to make the thermal of methyl alcohol visible, but the outline was less sharp than with the white precipitate. Only one upward moving case (no. 201) was measured, and this agrees closely with the others.

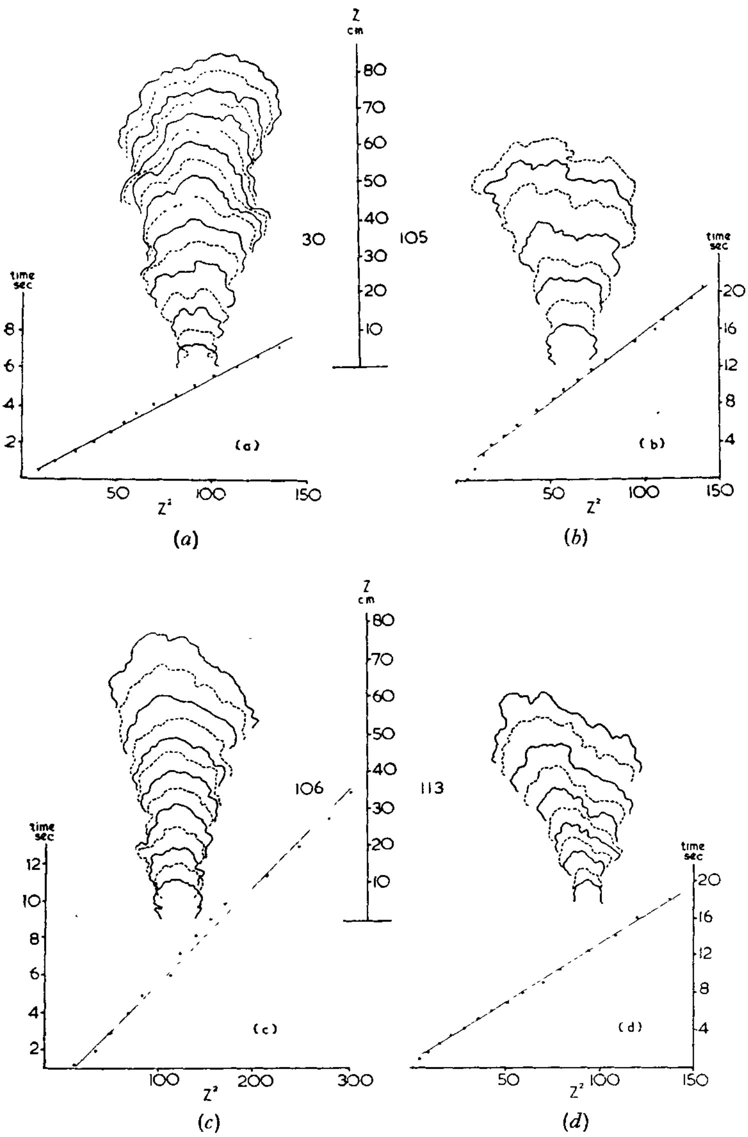

Figure 5. Successive outlines of thermals traced from photographs. Beneath each is a graph of z^2 against t. (*a*) and (*b*) show fairly typical thermals; 106 (*c*) is an example of a very large value of n; 113 (*d*) shows a very asymmetrical thermal.

In figure 6 we have plotted $(V_0 B_0)^{1/2}$ against k^{-1}, and the straight line was drawn by eye through the origin to fit the points. There is no evident systematic deviation from this line and so viscosity can reasonably be assumed to be negligible. The straight line represents the relation $k^{-1} = 180(V_0 B_0)^{1/2}$, i.e. (see (7))

$$2nC/m^{1/2} = 180g^{1/2} = 5{\cdot}7. \tag{8}$$

It was observed that n and m were not the same in all thermals, rough mean values being

$$m = 3, \qquad n = 4. \tag{9}$$

The variations in m did not appear to be related to other properties of the thermals, but some systematic variation in n was detected (see below). With these values we find

$$C \doteqdot 1{\cdot}2. \tag{10}$$

Experiment no.	$(V_0 B_0)^{1/2}$ $\mathrm{cm}^{3/2}$	k^{-1} $\mathrm{cm}^2\,\mathrm{sec}^{-1}$	$(V_0 B_0)^{-1/2}k^{-1}$	n	$(V_0\,B_0)^{-1/2}k^{-1}n^{-1/2}$
25	6·63	1140	170	3·8	87
110	6·53	1390	210	4·8	96
107	6·31	1090	170	3·7	89
112	5·81	1320	220	4·8	100
104	5·49	1010	200	4·1	99
102	4·97	1000	200	4·1	99
109	4·54	880	190	4·2	93
24	4·45	850	190	3·6	100
72	3·86	625	160	3·1	91
111	3·45	568	160	3·0	93
30	2·82	490	170	3·3	93
106	—	477	—	5·0	—
108	2·44	406	170	3·5	91
118	1·94	357	190	3·6	100
201	1·47	250	170	4·8	78
117	1·37	250	180	4·0	90
113	1·26	200	150	2·9	88
105	1·01	172	170	2·9	100

Table 1. Measurements of total mass (deficiency or excess), k (giving the velocity as a function of height), and rate of widening n.

6. Variations in the angle of the enveloping cone

We may attempt to take account of the variations of n in the following way. If the volume is expressed in terms of r^2z, and suffix 1 is used to denote values at a given time, then

$$\overline{B}r^2z = \overline{B}_1\,r_1^2\,z_1. \tag{11}$$

In the place of (2), we have

$$w = Cn^{-1/2}(g\overline{B}z)^{1/2} = Cn^{1/2}(g\overline{B}_1\,r_1^2\,z_1)^{1/2}z^{-1}. \tag{12}$$

Accordingly instead of having C as a constant we might have $Cn^{1/2}$ as a constant, for now $g\overline{B}_1\,r_1^2\,z_1$ represents the weight (deficiency or excess) of the thermal. In that case

$$\tfrac{1}{2}m^{1/2}(gB_0\,V_0)^{-1/2}k^{-1}n^{-1/2} = Cn^{1/2} = \text{const.} \tag{13}$$

To test this we have plotted $(V_0 B_0)^{-1/2}k^{-1}$ against n in figure 7, and the curve drawn is part of the parabola represented by

$$n = 0{\cdot}00012(V_0 B_0)^{-1}k^{-2}. \tag{14}$$

If the points lay on this curve it would imply that $Cn^{1/2}$ was in fact constant. This appears to take some account of the variations in n, because over this series of experiments $Cn^{1/2}$ varies by only half as much as C itself (see table 1); we have no indication of the mechanism, except that if the thermal grows along a wider cone it travels more slowly at a given distance from the virtual origin, presumably because it has been more diluted. If $m = 3$, equations (13) and (14) give

$$Cn^{1/2} = 2{\cdot}5. \tag{15}$$

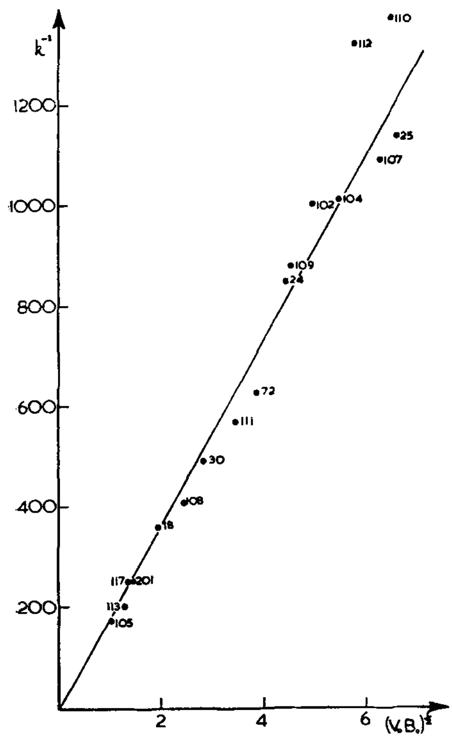

Figure 6.

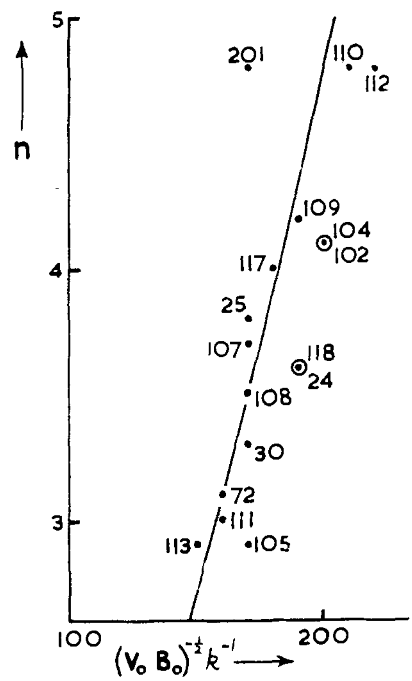

Figure 7.

Figure 6. $(V_0 B_0)^{1/2}$ plotted against k^{-1} for the thermals enumerated in table 1. The *straight line represents $k^{-1} = 180\ (V_0 B_0)^{1/2}$.*

Figure 7. The relationship $n = 0{\cdot}00012/V_0 B_0 k^2$, indicated by the curve drawn, is an improvement upon the crude assumption that n is the same for all experiments.

The tendency of a thermal to keep the same value of n throughout its life may be compared with the behaviour of the air bubbles of Davies and Taylor whose angular 'aperture' varied from one bubble to another.

In general, the wider thermals (smaller n) tended to leave more 'debris' behind and this may possibly account for their slower motion. The unusually slow one (113, shown in figure 5 (d)) developed very asymmetrically and only part of it was in effect measured, which would imply that its total buoyancy was overestimated. The point derived from it lies below the line in figure 6.

7. Comparison with atmospheric thermals

The only published measurements of thermals in the atmosphere available for comparison are those given by Malkus & Scorer (1955). They measured the rate of rise of isolated cumulus towers, and found that early in their life on emerging from the parent cloud the relationship

$$w^2 = \tfrac{8}{9} g\overline{B}r \tag{16}$$

was fairly well obeyed. The cloud towers ceased to rise after ascending about 1·5 diameters, mainly because the buoyancy decreased to zero, or even changed sign, as the cloud droplets evaporated when the thermal was mixed with the surrounding dry air. In 1·5 diameters the whole of the interior of the thermal would have been exposed to mixing. Inside the parent cloud there is no such evaporation, so that soon after emerging the motion would have been little affected by evaporation in the exterior shell of the thermal.

In the light of the present experiments and the nature of the mixing process it is probable that their estimates of buoyancy were too high; hence the value $C = 8/9$ obtained for clouds by comparing (2) and (16) would be expected to be a little below the true value. This agrees satisfactorily with the value of 1·2 obtained from the present experiments, and is certainly within experimental error. Of course not all differences should be attributed to such errors. It should be pointed out that the cloud thermals were ascending into stably stratified surroundings and were not increasing in size as they ascended, so that the terminal velocity may have differed on this account; but the difference cannot have been very great because the thermals were decelerating. The measurement of r in the case of the clouds was made soon after emergence from the parent cloud.

Recently Ludlam & Saunders (1956) have made measurements of the vertical velocity of cumulus towers just as they began to emerge as thermals from the parent cloud. They did not attempt to measure the diameter but, having (12) in mind, expressed the size simply as distance above the ground. It cannot be much in error to assume that the virtual source is at the ground. Consequently it was possible to make an estimate of the 'constant' $Cn^{-1/2}$ in (12), and the mean value they obtained was roughly 0·5. This was based on the behaviour of the most rapidly rising thermals at any moment and not on the motion of many slower ones which could have originated at higher levels with unknown z. The closeness of agreement with the value of 0·6 obtained in our experiments is certainly fortuitous in view of the roughness of the various estimates that are necessary, but it encourages the belief that no new dynamical process is involved even though the linear scale is much greater. Their calculations are not given in the paper referred to, other aspects of the observations being discussed there, but they will be submitted for publication in due course.

An attempt was made during the summer of 1956 by Miss Betsy Woodward to make the appropriate measurements in clear air thermals below cloud base in a Skylark II glider. The humidity and temperature inside and outside of a thermal were measured using a wet and dry thermistor out-of-balance resistance thermometer, the readings being spoken into a wire recorder during flight. Spot readings had to be used to determine mean buoyancy of the thermal. The size of the thermal was estimated very roughly from the size of the turning circle which she could use during the ascent. The vertical velocity is not easily measured because it varies from one part of the thermal to another. It can be properly measured only if the glider has risen through the thermal to near the top, where, because of the decrease of vertical velocity and the outward radial component (which is equivalent to a downward velocity for a circling glider) the glider can rise only as fast as the whole thermal.

Using the observed buoyancies, etc. (Woodward 1956), a vertical velocity of 2 m/sec was deduced using a value of 1·2 for C and $r = 160$ m, $\bar{B} = \frac{1}{540}$, but since the uncertainties are so many it is neither surprising nor gratifying that the observed rate of rise of the glider when near the top of the thermal was only 1 m/sec. It is not considered profitable to attempt to explain away this discrepancy because there are too many different ways in which to do it easily. The observation is quoted because it shows that the model experiments again lead to an estimate of w of the correct order of magnitude.

8. The interior motion of a thermal

Miss Woodward has made the interior motion visible by releasing a transparent thermal containing white pellets about 5 mm in diameter balanced with pieces of fine wire attached to them so that they had a very small terminal velocity in water. The thermal was then illuminated by a flat beam of light about 3 cm thick, and the motion in the central vertical section of the thermal was photographed on 16 mm cine film.

The positions of particles within the thermal in successive pictures were drawn by steadily reducing the picture of the thermal to a constant size on a screen as the motion proceeded. The motion during the whole life of the thermal was thus concentrated into one picture, on the assumption that the motion was similar at all stages. Isopleths of horizontal and vertical component of velocity obtained in this way are shown in figure 8. Speeds are expressed as multiples of the forward velocity of the front of the thermal. This figure is based on observation of only one thermal.

It is seen that the velocity is a maximum in the centre, whereas in a vortex in which the vorticity is concentrated in a ring the velocity is a maximum at the inner surface of the ring and decreases towards the axis.

9. Comparison with buoyant vortex rings

In a recent paper Turner (1957) has discussed the behaviour of buoyant fluid ejected in the form of a ring vortex. Several years ago some glider pilots were accustomed to think of the thermals in which they soared as possessing a configuration like a ring vortex. We now see that there is much justification for this. One important difference is that a thermal is not possessed of any circulation, kinetic energy, or forward momentum at the moment of release. The circulation is generated gradually in the early stages of its existence. As Dr Batchelor noted to me in correspondence, the circulation round a thermal when it has achieved the final configuration is proportional to wz and is therefore constant (see (2), (3) and (5)). In

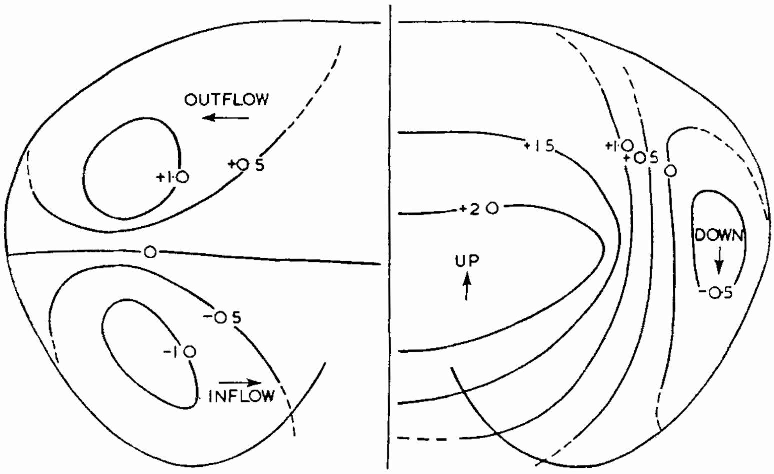

Figure 8. The distribution of velocity obtained by observing the motion of particles inside a liquid thermal. The values of the horizontal (left hand diagram) and vertical (right) velocities are expressed as multiples of the vertical velocity of the front of the thermal.

the early stages there is a hydrostatic buoyancy force acting on a column of buoyant fluid up the centre and increasing the circulation. Later on much of this column is replaced by exterior fluid, as is seen in figure 9 (plate 2). If the rate of change of circulation is zero, then

$$\oint \frac{dp'}{\rho} = 0 \tag{17}$$

for a circuit passing through the centre of the thermal and round the outside, p' being the departure of the pressure from the hydrostatic value in the exterior fluid. If the fluid is buoyant along any part of the axis of the thermal the tendency to produce circulation must be counteracted by departure from the hydrostatic pressure due to the motion. Since the motion has been produced by the buoyancy forces we should expect the configuration of buoyancy to change to a distribution which does not continue to produce circulation; this means reducing the depth of buoyant fluid up the axis of the thermal.

In Turner's vortex rings the circulation remained constant because the buoyant fluid did not extend to the centre. The initial impulse was of the order of 1500 gm cm sec^{-1}. The buoyancy force was of the order of 300 dynes and the impulse of a ring was increased by a factor of between 2 and 10 during the period of observation. One would suspect that ultimately its motion would tend towards that of an isolated thermal as the initial impulse tended towards a small fraction of the total, but the rings did not reach this stage in his experiments.

The only constant characteristics of a thermal are its total weight (deficiency or excess), $g\bar{B}V$, and the fluid density ρ. Dimensional considerations show that its circulation K must therefore be given by

$$K = \mathrm{const}(g\bar{B}V/\rho)^{1/2}. \qquad (18)$$

It seems that if the circulation is less than this value the buoyancy forces create circulation. But if the circulation is much in excess of this value mixing towards the axis must be inhibited. The vortex ring could only become a thermal if the circulation is reduced to the value appropriate to its total buoyancy according to (18). This might happen ultimately on account of viscosity, but then the viscous forces would interfere with the motion of the thermal. In Turner's case the total buoyancy of the rings was about 0·3 gm wt., which is much less than in the present experiments, in which it ranged from 1 to 40 gm wt., so that they are not immediately comparable.

Among those who have participated in this work Miss Betsy Woodward deserves special thanks for her assistance in performing the experiments and measuring the photographs. She has also done much pioneer work in the investigation of thermals in gliders. Her work is at present supported by the Munitalp Foundation.

References

Batchelor, G. K. 1954 Heat convection and buoyancy effects in fluids, *Quart. J. Roy. Met. Soc.* **80**, 339.

Davies, R. M. & Taylor, G. I. 1950 The mechanics of large bubbles rising through extended liquids and through liquids in tubes, *Proc. Roy. Soc.* A, **200**, 375.

Ludlam, F. H. & Saunders, P. M. 1956 Shower formation in large cumulus, *Tellus* **8**, 424.

Ludlam F. H. & Scorer, R. S. 1953 Convection in the atmosphere, *Quart. J. Roy. Met. Soc.* **79**, 317.

Malkus, J. S. & Scorer, R. S. 1955 The erosion of cumulus towers, *J. Met.* **12**, 43.

Morton, B. R., Taylor, G. I. & Turner, J. S. 1956 Turbulent gravitational convection from maintained and instantaneous sources, *Proc. Roy. Soc.* A, **234**, 1.

Scorer, R. S. & Ronne, C. 1956 Experiments with convection bubbles, *Weather* **11**, 151.

Turner, J. S. 1957 Buoyant vortex rings, *Proc. Roy. Soc.* A, **239**, 61.

Welch, A., Welch, L., & Irving, F. G. 1955 *The Soaring Pilot.* London: John Murray.

Woodward, B. 1956 Exploration by sailplanes of the structure of thermals, *O.S.T.I.V. News*; Supplement to *Swiss Aero. Revue*, Sept., 1956.

Erratum

The first equation in the Summary should read as $z = nr$.